Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Name: ______________________________

Numerical Python Pre-requisites

1. In these questions, you will work with the Numerical Python library, `numpy`. NumPy is the fundamental package for scientific computing in Python; however, we will first motivate the need for such a package by examining the Python `list` type.

5 (a) Create a list called `x` containing the elements 2, 3, 1, and 0.

5 (b) Create a list called `y` containing the elements 4, 1, 4, and 1.

5 (c) Add the lists `x` and `y` to obtain `z` (assume that `x` and `y` have the same number of elements.[1])

[1] A fancy solution is to use `zip` (`http://docs.python.org/library/functions.html#zip`, but otherwise you will need to use a clever for loop.

2. We will now perform similar operations, but using tuples. In the past, you have used tuples to store the positions of objects.

5 (a) Create a position tuple `t` at $(3, 5)$ and `u` at $(1, 2)$.

5 (b) Add the tuples `t` and `u` element-wise to obtain `v`.

Numerical Python

3. In these questions, we will see how `NumPy` can help with with these routine transformations.

5 (a) Import the `numpy` module, but for convenience, alias it to `np`.

5 (b) Convert the list `x` above to a `numpy` array, and call it `xx`.

5 (c) Convert the list `[6,3,5,5]` to a `numpy` array, called `yy`.

5 (d) Add the arrays (vectors) `xx` and `yy` element-wise to obtain `zz`.

4. We will now perform similar operations using tuples.

5 (a) Convert the tuples `t` and `u` (from the previous question) into `numpy` Arrays `tt` and `uu`.

5 (b) Add the vectors `tt` and `uu`.

Newtonian Physics

5. Given the one-dimensional kinematic equations:

$$p \leftarrow p_o + v_o t + \frac{1}{2} a t^2 \tag{1}$$

$$v \leftarrow at \tag{2}$$

5 (a) Compute p when $p_o = 3$, $v_o = 1$, $a = 2$, and $t = 0.2$.

5 (b) Compute v given the parameters of the previous part.

6. When time intervals are small (as in the case when the frame rate is high), the contribution of acceleration to position will be tiny. In this case, we can use the Newton-Euler-1 integration update approximation. Given the equations:

$$p \leftarrow p_o + v_o t \tag{3}$$

and

$$v \leftarrow at \tag{4}$$

5 (a) Compute p when $p_o = 3$, $v_o = 1$, $a = 2$, and $t = 0.2$.

(b) Compute v given the parameters of the previous part.

Vectors

10 7. Draw a Cartesian coordinate system and plot the following vectors from $(0,0)$: $\vec{a} = \begin{bmatrix}2\\3\end{bmatrix}$, $\vec{b} = \begin{bmatrix}-3\\-2\end{bmatrix}$, $\vec{c} = \begin{bmatrix}1\\3\end{bmatrix}$, $\vec{d} = \begin{bmatrix}3\\0\end{bmatrix}$, and $\vec{e} = \begin{bmatrix}0\\4\end{bmatrix}$

8. Perform the following vector operations:

5 (a)

$$\begin{bmatrix}5\\2\end{bmatrix} \cdot 4 \tag{5}$$

5 (b)

$$\begin{bmatrix}-3\\-2\end{bmatrix} - \begin{bmatrix}3\\-2\end{bmatrix} \tag{6}$$

5 (c)

$$\begin{bmatrix} 4 \\ -3 \end{bmatrix} + 2 \cdot \begin{bmatrix} 1 \\ 3 \end{bmatrix} \tag{7}$$

5 (d) The dot product operation takes two vectors and returns a scalar.

$$\begin{bmatrix} 3 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -4 \end{bmatrix} \tag{8}$$

9. Assume that your opponent is at $\vec{t} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ from you.

5 (a) At what point (x, y) is your opponent if you are at $(3, 4)$?

5 (b) Compute the Euclidean distance to the opponent. Recall that the distance formula is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{9}$$

5 (c) Using $\vec{t}$, compute

$$|\vec{t}| = \sqrt{\vec{t} \cdot \vec{t}} \tag{10}$$

10 (d) Re-write the above equation using `numpy`. The `numpy` function for dot product is `np.dot`.

Normalization

5 10. Assume that you are at location $(3, 2)$. You now move using the vector $\vec{t} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, then using $\vec{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. What is your destination point?

5 11. Plot these vectors on a graph.

12. Assume that you are once again at location $(3, 2)$. You now move using the vector $\vec{r} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$. What is your destination? Plot this vector on the graph above.

13. Assume that you are at $(3, 2)$ at that your opponent is at $(5, 6)$.

5 (a) What is the vector to get from you to the opponent? (Hint: the arrow head is the first coordinate).

5 (b) What is the vector to get from your opponent to you?

5 (c) Given the vector $\vec{v} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, what is $|\vec{v}|$?

5 (d) Using $\vec{v}$, compute the normalized vector:

$$\hat{v} = \frac{\vec{v}}{|\vec{v}|}$$

(e) What is the length of $\hat{v}$, that is $|\hat{v}|$ (A normalized vector is also known as a unit vector)?

Question	Points	Score
1	15	
2	10	
3	20	
4	10	
5	10	
6	5	
7	10	
8	20	
9	25	
10	5	
11	5	
12	0	
13	20	
Total:	155	